DEPARTMENT OF COMPUTER SCIENCE
FACULTY OF BASIC AND APPLIED SCIENCES
ARTHUR JAVIS UNIVERSITY, AKPABUYO
COURSE CODE: STA112

COURSE TITLE: PROBABILITY 1 **2 Units**

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Concepts and principles of Probability. Permutation and Combination. Random variables. Probability and distribution Functions. Basic distributions: Bernoulli, Binomial, Hypergeometric, Poisson and Normal.

MODULE 1

INTRODUCTION

The probability of something happening is the likelihood or chance of it happening. Values of probability lie between 0 and 1, where 0 represents an absolute impossibility and 1 represents an absolute certainty. The probability of an event happening usually lies somewhere between these two extreme values and is expressed either as a proper or decimal fraction. Examples of probability are:

that a fair, six-sided dice will stop with a 3 upwards is

$$1/6 \text{ or } 0.1667$$

that a fair coin will land with a head upwards

$$1/2 \text{ or } 0.5$$

that a student will pass an exam

$$1/2 \text{ or } 0.5$$

that a student will fail an exam

$$1/2 \text{ or } 0.5$$

If p is the probability of an event happening and q is the probability of the same event not happening, then the total probability is p+q and is equal to unity (1), since it is an absolute certainty that the event either does or does not occur,

i.e. $p + q = 1$

SOME DEFINITIONS

1. **Expectation**

The **Expectation**, E, of an event happening is defined in general terms as the product of the probability p of an event happening and the number of attempts made, n,

i.e. **E = pn**

Thus, since the probability of obtaining a 3 upwards when rolling a fair dice is $^1/_6$, the expectation of getting a 3 upwards on four throws of the dice is $^1/_6$ x 4, i.e. $^2/_3$

Thus expectation is the average occurrence of an event.

2. **Dependent event**

A dependent event is one in which the probability of an event happening affects the probability of another ever happening.

Let 5 best students be taken at random from a batch of 100 students in AJU for a competition in UniCal, and the probability of them being successful in the competition, p1, be determined. At some later time, let another 5 students be taken at random from the 95 remaining students in the batch and the probability of them being successful also,p2, be determined. The value of p2 is different from p1since batch size has effectively been altered from 100 to 95, i.e. probability p2 is dependent on probability p1. Since students are drawn, and then another 5 students drawn without replacing the first 5, the second random selection is said to be **without replacement**.

3. **Independent event**

An independent event is one in which the probability of an event happening does not affect the probability of another event happening.

If 5 best students are taken at random from a batch of the 100 students and the probability of them being successful p1 is determined and the process is repeated after the original 5 have been replaced in the batch to give p2, then p1 is equal to p2. Since the 5 students are replaced between draws, the second selection is said to be **with replacement.**

4. **Mutually Exclusive event**

LAWS OF PROBABILITY

The addition law of probability

The addition law of probability is recognised by the word ‘or’ joining the probabilities. If P_A is the probability of event A happening and P_B is the probability of event B happening, the probability of event A or event B happening is given by

P_A+P_B.

Similarly, the probability of events A or B or C or...N happening is given by

$P_A+P_B+P_C+\cdots+P_N$

The multiplication law of probability

The multiplication law of probability is recognised by the word ‘and’ joining the probabilities. If P_A is the probability of event A happening and P_B is the probability of event B happening, the probability of event A and event B happening is given by

$P_A \times P_B$.

Similarly, the probability of events A and B and C and...N happening is given by

$P_A \times P_B \times P_C \times \cdots \times P_N$

Worked problems on probability

Problem1. Determine the probabilities of selecting at random (a) a man, and (b) a woman from a crowd of 20 men and 33 women.

Solution

(a) The probability of selecting at random a man, p, is given by

the ratio $\frac{number\ of\ men}{number\ of\ crowd}$

i.e. $p = \frac{20}{20+33} = \frac{20}{53}$

or 0.3774

(b) The probability of selecting at random a women, q, is given by

the ratio $\frac{number\ of\ women}{number\ of\ crowd}$

i.e. $q = = \frac{33}{20+33} = \frac{33}{53} =$

or 0.6226

(Check: the total probability should be equal to 1;

$p=\frac{20}{53}$ and $q = \frac{33}{53}$

thus the total probability,

$p + q = \frac{20}{53} + \frac{33}{53} = \frac{53}{53} = 1$

hence no obvious error has been made).

Problem 2. Find the expectation of obtaining a 4 upwards with 3 throws of a fair dice.

Solution

Expectation is the average occurrence of an event and is defined as the probability times(x) the number of attempts. The probability, p, of obtaining a 4 upwards for one throw of the dice is $\frac{1}{6}$

Also, 3 attempts are made, hence n=3 and the expectation, E, is pn,

i.e. E=.$\frac{1}{6}$ x 3 = $\frac{1}{2}$ or 0.50

Problem 3. If there are 10 students offering sta112, Calculate the probabilities of selecting at random:

(a) the best student in the class

(b) the first and second best student in both the first and second selection with replacement

Solution

(a) Since only one of the ten students can emerge as the best, the probability of

Selecting at random the best student is

$$\frac{number\ of\ best\ student}{number\ of\ students}$$

i.e. $\frac{1}{10}$ or 0.10

(b) The probability of selecting best student in the first

Selection is $\frac{1}{10}$

The probability of selecting the second best in the second selection with replacement is

$$\frac{1}{10}$$

The probability of selecting the best first and second students in the first and second selection is given by the multiplication law of probability, i.e. probability = $\frac{1}{10}$ x $\frac{1}{10}$ =

$\frac{1}{100}$ or 0.01

Problem 4. The probability of a component failing in one year due to excessive temperature is $\frac{1}{20}$, due to excessive vibration is $\frac{1}{25}$, and due to excessive humidity is $\frac{1}{50}$.

Determine the probabilities that during a one-year period a component: (a) fails due to excessive temperature and excessive vibration, (b) fails due to excessive vibration or excessive humidity, and (c) will not fail because of both excessive temperature and excessive humidity.

<u>Solution</u>

Let P_A be the probability of failure due to excessive temperature,

then $P_A = \frac{1}{20}$ and

the probability of not failing, $\overline{Pa} = \frac{19}{20}$

Let P_B be the probability of failure due to excessive vibration, then

$P_B = \frac{1}{25}$ and the probability of not failing

$\overline{Pb} = \frac{24}{25}$

Let P_C be the probability of failure due to excessive humidity,

Then $P_C = \frac{1}{50}$ and

$\overline{Pc} = \frac{49}{50}$

(a) The probability of a component failing due to excessive temperature and excessive vibration is given by:

$P_A \times P_B = \frac{1}{20} \text{ x } \frac{1}{25}$

$= \frac{1}{500}$ or 0.002

(b) The probability of a component failing due to excessive vibration or excessive humidity is:

$P_B + P_C = \frac{1}{25} + \frac{1}{50}$

$= \frac{3}{50}$ or 0.06

(c) The probability that a component will not fail due excessive temperature and will not fail due to excess humidity is:

$$P_A \times P = \frac{19}{20} \times \frac{49}{50}$$
$$= \frac{931}{1000} \text{ or } 0.931$$

Problem 5. A batch of 100 capacitors contains 73 which are within the required tolerance values, 17which are below the required tolerance values, and the remainder are above the required tolerance values. Determine the probabilities that when randomly selecting a capacitor and then a second capacitor: (a) both are within the required tolerance values when selecting with replacement, and (b) the first one drawn is below and the second one drawn is above the required tolerance value, when selection is without replacement.

Solution

(a) The probability of selecting a capacitor within the required tolerance values

$$\text{is } \frac{73}{100}.$$

The first capacitor drawn is now replaced and a second one is drawn from the batch of 100.

The probability of this capacitor being within the required tolerance values is also $\frac{73}{100}$

Thus, the probability of selecting a capacitor within the required tolerance values for both the first and the second draw

$$\text{is } \frac{73}{100} \text{ x } \frac{73}{100} =$$

$$\frac{5329}{10000} = 0.5329$$

(b) The probability of obtaining a capacitor below the required tolerance values on the first draw is $\frac{17}{100}$

There are now only 99 capacitors left in the batch, since the first capacitor is not replaced. The probability of drawing a capacitor above the required tolerance values on the second draw is $\frac{10}{99}$, since there are (100−73−17), i.e. 10 capacitors above the required tolerance value. Thus, the probability of randomly selecting a capacitor below the required tolerance values and followed by randomly selecting a capacitor above the tolerance values is

$$\frac{17}{100} \text{ x } \frac{10}{99} = \frac{170}{9900} = \frac{17}{990} \text{ or } 0.0172$$

Exercise

1. In a batch of 45 lamps there are 10 faulty lamps. If one lamp is drawn at random, find the probability of it being

(a) faulty and (b) satisfactory.

2. A box of fuses are all of the same shape and size and comprises 23 2A fuses, 47 5A fuses and 69 13A fuses.

Determine the probability of selecting at random (a) a 2A fuse, (b) a 5A fuse and (c) a 13A fuse.

3. (a) Find the probability of having a 2 upwards when throwing a fair 6-sided dice. (b) Find the probability of having a 5 upwards when throwing a fair 6-sided dice. (c) Determine the probability of having a 2 and then a 5 on two successive throws of a fair 6-sided dice.

4. Student A and B are good friends, after first semester, the probability of student A graduating in record time is $\frac{3}{5}$ and the probability of student B graduating in record time is $\frac{2}{5}$. Calculate the probabilities of (a) both A and B graduating at the same time, (b) only student A graduating, i.e. student A graduating and student B not graduating, (c) only student B graduating, and (d) either A, or B, or A and B graduating.

5. When testing 1000 soldered joints, 4 failed during a vibration test and 5 failed due to having a high resistance. Determine the probability of a joint failing due to (a) vibration, (b) high resistance, (c) vibration or high resistance and (d) vibration and high resistance.

<u>Solutions</u>

1.(a) 2/9 or 0.2222 (b) 7/9 or 0.7778

2.(a)23/139 or 0.1655 (b) 47/ 139 or 0.3381 (c) 69/139 or 0.4964

3.(a) 1/6 (b)1/6 (c) 1/36

4.(a) 2/5 (b) 1/5 (c) 4/15 (d) 13/15

5.(a) 1/250 (b) 1/200 (c) 9/1000 (d) 1/50000

Problem 6. A batch of 40 components contains 5 which are defective. A component is drawn at random from the batch and tested and then a second component is drawn.

Determine the probability that neither of the components is defective when drawn (a) with replacement, and (b) without replacement.

<u>Solution</u>

(a) With replacement

The probability that the component selected on the first draw is satisfactory is

$$\frac{35}{40} = \frac{7}{8}$$

The component is now replaced and a second draw is made. The probability that this component is also satisfactory is $\frac{7}{8}$

. Hence, the probability that both the first component drawn and the second component drawn are satisfactory

$$\text{is: } \frac{7}{8} \times \frac{7}{8} = \frac{49}{64} = 0.7656$$

(b) Without replacement

The probability that the first component drawn is satisfactory is $\frac{7}{8}$

. There are now only 34 satisfactory components left in the batch and the batch number is 39. Hence, the probability of drawing a satisfactory component on the second draw is

$$\frac{34}{39}$$

. Thus the probability that the first component drawn and the second component drawn are satisfactory, i.e. neither is defective, is:

$$\frac{7}{8} \times \frac{34}{39} = \frac{238}{312} \text{ or } 0.7628$$

Problem 7. A batch of 40 components contains 5 which are defective. If a component is drawn at random from the batch and tested and then a second component is drawn at random, calculate the probability of having one defective component, both with and without replacement.

Solution

The probability of having one defective component can be achieved in two ways. If p is the probability of drawing a defective component and q is the probability of drawing a satisfactory component, then the probability of having one defective component is given by drawing a satisfactory component and then a defective component or by drawing a defective component and then a satisfactory one, i.e. by $q \times p + p \times q$

With replacement:

$$P = \frac{5}{40} = \frac{1}{8} \text{ and } q = \frac{35}{40} = \frac{7}{8}$$

Hence, probability of having one defective component is:

$$\frac{1}{8} \text{ x } \frac{7}{8} + \frac{7}{8} \text{ x } \frac{1}{8}$$

$$\text{i.e. } \frac{7}{64} + \frac{7}{64} = \frac{7}{32} \text{ or } 0.2188$$

Without replacement:

$p1 = \frac{1}{8}$ and $q1 = \frac{7}{8}$ on the first of the two draws. The batch number is now 39 for the second draw, thus,

$$p2 = \frac{5}{39} \text{ and } q2 = \frac{35}{39}$$

$$p1q2 + q1p2 = \frac{1}{8} \times \frac{35}{39} + \frac{7}{8} \times \frac{5}{39} = \frac{35+35}{312} = \frac{70}{312} \text{ or } 0.2244$$

Problem 8. A box contains 74 brass washers, 86 steel washers and 40 aluminium washers. Three washers are drawn at random from the box without replacement. Determine the probability that all three are steel washers

Solution

Assume, for clarity of explanation, that a washer is drawn at random, then a second, then a third (although this assumption does not affect the results obtained).The total number of washers is 74+86+40, i.e. 200.

The probability of randomly selecting a steel washer on the first draw is

$$\frac{86}{200}$$

. There are now 85 steel washers in a batch of 199. The probability of randomly selecting a steel washer on the second draw is

$$\frac{85}{199}$$

There are now 84 steel washers in a batch of 198. The probability of randomly selecting a steel washer on the third draw is

$$\frac{84}{198}.$$

Hence the probability of selecting a steel washer on the first draw and the second draw and the third draw is:

$$\frac{86}{200} \times \frac{85}{199} \times \frac{84}{198} = \frac{614\,040}{7880400} = 0.0779$$

Problem 9. For the box of washers given in Problem 8 above, determine the probability that there are no aluminium washers drawn, when three washers are drawn at random from the box without replacement.

The probability of not drawing an aluminium washer on the first draw

is $1-\left(\frac{40}{200}\right)$, i.e. $\frac{160}{200}$.

There are now 199 washers in the batch of which 159 are not aluminium washers. Hence, the probability of not drawing an aluminium washer on the second draw

Is $\frac{159}{199}$. Similarly, the probability of not drawing an aluminium washer on the third draw is $\frac{158}{198}$. Hence the probability of not drawing an aluminium washer on the first and second and third draws is

$$\frac{160}{200} \times \frac{159}{199} \times \frac{158}{198} = \frac{4019520}{7880400} = 0.5101$$

Problem 10. For the box of washers in Problem 8 above, find the probability that there are two brass washers and either a steel or an aluminium washer when three are drawn

at random, without replacement.

Two brass washers (A) and one steel washer (B) can be obtained in any of the following ways:

1st draw	2nd draw	3rd draw
A	A	B
A	B	A
B	A	A

Two brass washers and one aluminium washer (C) can also be obtained in any of the following ways:

1st draw	2nd draw	3rd draw
A	A	C
A	C	A
C	A	A

Thus there are six possible ways of achieving the combinations specified. If A represents a brass washer, B a steel washer and C an aluminium washer, then the combinations and their probabilities are as shown:

DRAWS			PROBABILITIES
First	Second	Thrid	
A	A	B	$\frac{74}{200} \times \frac{73}{199} \times \frac{86}{198} = 0.0590$
A	B	A	$\frac{74}{200} \times \frac{86}{199} \times \frac{73}{198} = 0.0590$
B	A	A	$\frac{86}{198} \times \frac{74}{200} \times \frac{73}{199} = 0.0590$
A	A	C	$\frac{74}{200} \times \frac{73}{199} \times \frac{40}{198} = 0.0274$
A	C	A	$\frac{74}{200} \times \frac{40}{199} \times \frac{73}{198} = 0.0274$
C	A	A	$\frac{40}{198} \times \frac{74}{200} \times \frac{73}{199} = 0.0274$

The probability of having the first combination or the second, or the third, and so on, is given by the sum of the probabilities,

i.e. by $3 \times 0.0590 + 3 \times 0.0274 = 0.2592$

Problem 11. For the box of washers in Problem 8 above, find the probability that there is an aluminium washer when three are drawn at random, without replacement.

There are 9 possible ways of achieving the combinations specified. If A represents a brass washer, B a steel washer and C an aluminium washer, then the combinations and their probabilities are as shown:

DRAWS			PROBABILITIES
First	Second	Thrid	
A	A	C	$\frac{74}{200} \times \frac{73}{199} \times \frac{40}{198} = 0.0274$
A	C	A	$\frac{74}{200} \times \frac{40}{199} \times \frac{73}{198} = 0.0274$
C	A	A	$\frac{40}{200} \times \frac{74}{199} \times \frac{73}{198} = 0.0274$
B	B	C	$\frac{86}{200} \times \frac{85}{199} \times \frac{40}{198} = 0.0371$
B	C	B	$\frac{86}{200} \times \frac{40}{199} \times \frac{85}{198} = 0.0371$
C	B	B	$\frac{40}{200} \times \frac{86}{199} \times \frac{85}{198} = 0.0371$
A	B	C	$\frac{74}{200} \times \frac{86}{199} \times \frac{40}{198} = 0.03230$
B	C	A	$\frac{68}{200} \times \frac{40}{199} \times \frac{74}{198} = 0.03230$
C	A	B	$\frac{40}{200} \times \frac{74}{199} \times \frac{86}{198} = 0.03230$

The probability of having one aluminium represented by C is obtained in either of the first combination or the second, or the third, and so on, is given by the sum of the probabilities,

i.e. by 3×0.0274 +3×0.0371 + 3x0.03230 = 0.2592

0.0822 + 0.1113 + 0.0969 =0.2904

Exercise

1. The probability that component A will operate satisfactorily for 5 years is 0.8 and that B will operate satisfactorily over that same period of time is 0.75. Find the probabilities that ina5yearperiod: (a) both components operate satisfactorily, (b) only component A will operate satisfactorily, and (c) only component B will operate satisfactorily.
2. In a particular street, 80%of the houses have telephones. If two houses selected at random are visited, calculate the probabilities that (a) they both have a telephone and (b) one has a telephone but the other does not have telephone.
3. Vero board pins are packed in packets of 20 by a machine. In a thousand packets, 40 have less than 20 pins. Find the probability that if 2 packets are chosen at random, one will contain less than 20 pins and the other will contain 20 pins or more.
4. A batch of 1 kW fire elements contains 16 which are within a power tolerance and 4which are not. If 3 elements are selected at random from the batch, calculate the probabilities that (a) all three are within the power tolerance and (b) two are within but one is not within the power tolerance.
5. An amplifier is made up of three transistors, A, B and C. The probabilities of A, B or C being defective are $\frac{1}{20}$, $\frac{1}{25}$ and $\frac{1}{50}$, respectively. Calculate the percentage of

amplifiers produced (a) which work satisfactorily and (b) which have just one defective transistor.

6. A box contains 14 40W lamps, 28 60W lamps and 58 25W lamps, all the lamps being of the same shape and size. Three lamps are drawn at random from the box, first one, then a second, then a third. Determine the probabilities of: (a) getting one 25W, one 40Wand one 60Wlamp, with replacement, (b) getting one 25W, one 40Wand one 60W lamp without replacement, and (c) getting either one 25W and two 40W or one 60W and two 40W lamps with replacement.

Solution

(1.)(a) 0.6 (b) 0.2 (c) 0.15 (2.)(a) 0.64 (b) 0.32 (3).0.0768 4.(a) 0.4912 (b) 0.4211 5.(a) 89.38% (b) 10.25% 6.(a) 0.0227 (b) 0.0234 (c) 0.0169

Assignment

1. Determine the probability of winning a prize in a lottery by buying 10 tickets when there are 10 prizes and a total of 5000 tickets sold.
2. A sample of 50 resistors contains 44 which are within the required tolerance value, 4 which are below and the remainder which are above. Determine the probability of selecting from the sample a resistor which is (a) below the required tolerance, and (b) above the required tolerance. Now two resistors are selected at random from the sample. Determine the probability, correct to3decimal places, that neither resistor is defective when drawn (c) with replacement, and (d) without replacement. (e) If a resistor is drawn at random from the batch and tested, and then a second resistor is drawn from those left, calculate the probability of having one defective component when selection is without replacement.

Test

1. A box contains 74 brass washers, 86 steel washers and 40 aluminium washers. three washers are drawn at random from the box. Determine the probability that (a) two of them are steel washers with replacement (b) one of them is aluminium washers without replacement